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Abstract— The idle speed control is one of the most generic and basic automotive control problems. In this paper, the engine idle speed control problem comes down to optimization problem with input and state constraints. The controller based the quasiinfinite horizon nonlinear model predictive control (NMPC) methodology is designed to regulate the engine speed as close as possible to a reference value. The simulation results show that the controller can improve the dynamic performance of system and good control effect can be obtained as well as satisfying the system constraints.

Index Terms—Engine idle speed control, system constraints, nonlinear model predictive control, terminal region.

# I. INTRODUCTION

The engine idle speed control is always a tough task in the engine control. On the average, vehicles consume about 30 percent of their fuel in city driving during idling, and it is expected that with increased traffic loads this percentage will further increase in the future. The automotive exhaust emissions CO and HC in idle state are about 70 percent of total emission of pollutant. Therefore, it is important to optimize vehicle and powertrain operations at idle, especially with respect to often-conflicting requirements of improved fuel economy, reduced emissions, guaranteed combustion stability, and good noise, vibration and harshness quality.

Idle speed control goal is to maintain the engine speed as close as possible to a reference engine speed despite load torque disturbances(i.e. the air conditioning system, the steering wheel servo-mechanism) and engagements and disengagements of the transmission occurring when the driver operates on the clutch. In order to achieve the best fuel economy, the reference engine speed is chosen at the minimum value that yields acceptable combustion and emission quality, and noise, vibration and harshness characteristics [1] [2].

So far, most control techniques have been used for the engine idle speed control problem [1] [2] [3] [4], they include Multivariable control,  $l_1$  control,  $H_{\infty}$  control,  $\mu$ -synthesis, sliding mode control and LQ-based optimization.

In recent years, model predictive control (MPC) which is an important control strategy, has been intensively ap-

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plied to linear systems, nonlinear systems and hybrid systems [5] [6] [7] [8]. In general, systems in the process industry are inherently nonlinear, although linear models are widely used to solve control problems. In these case, linear models are obviously not adequate to describe the process dynamics and nonlinear models have to be used. This motivates the use of nonlinear model predictive control. NMPC is especially suited for the control of nonlinear multivariable systems subject to input and state constraints [9] [10]. The basic idea of NMPC is to solve at each time instant a finite horizon optimal control problem for the current state. The first part of the resulting open loop optimal control input is applied to the system until the next sampling instant, at which the finite horizon optimal control problem is solved again for the new state. Repeated application of this strategy results in a feedback law. However, the feedback law of NMPC does not naturally guarantee closed loop stability. The quasiinfinite horizon NMPC scheme in [7], which optimizes online an objective functional consisting of a finite horizon cost and a terminal cost subject to system dynamics, input constraints and an additional terminal state inequality constraint, successfully solves the stability problem of NMPC.

This paper mainly focuses on idle speed control problem of SI engine. The system model of the engine idle speed belongs to nonlinear model; Meanwhile, there inevitably exist input constraints (the spark advance angle is bounded to avoid knock (too much advance) and misfire (too little advance)) and state constraints (in order to avoid manifold pressure rising too much and to limit the range for safety reasons) in the system. Therefore, we utilize the quasi-infinite horizon NMPC methodology to deal with the idle speed control problem in this paper.

The paper is organized as follows. Section II gives the nonlinear model of engine idle speed system involving input and state constrains. Section III introduces the main idea of quasi-infinite horizon NMPC, which can deal with the problem of nonlinear and system constraints at the same time guaranteeing the asymptotic stability of the closed-loop system. And then gives the solving procedure for the algorithm of quasi-infinite horizon NMPC. Section IV translates the engine idle speed control problem into optimization problem with constraints, both analysis and simulation results are given and discussed in Subsection IV-B.

### **II. PROBLEM STATEMENT**

In this section, a nonlinear hybrid model of a 4-stroke 4cylinder spark ignition (SI) engine for idle speed control is briefly presented. Figure 1 shows the interactions between

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the three main subsystems which compose the engine: the intake manifold, the cylinders and the power-train.



### Fig. 1. The engine blocks

The pressure p of the intake manifold depends on the throttle valve angle  $\alpha$  and on the crankshaft revolution speed n. The manifold pressure determines the mass of air-fuel mixture m loaded by the cylinders. In each stroke of the 4-stroke engine cycle, the cylinders evolution is described in terms of the crankshaft angle  $\theta$ , which defines the location of the piston within the given stroke. The torque T generated by the cylinders depends on both the mass and the spark ignition time. Finally, the powertrain dynamics, controlled by the generated torque T, is subject to the sum of load torques  $T_l$  and depends on the position of the clutch. The gear is supposed to be in idle position.

The intake manifold is subject to the following dynamics:

$$\dot{p}(t) = a_p f_{out}(p(t), n(t)) + b_p S(\alpha(t))$$
(1)

where p is the manifold pressure (mbar) and n is crankshaft revolution speed (Revolutions Per Minute (RPM)),  $a_p$  and  $b_p$  are the manifold dynamic parameters,  $f_{out}$  represents the air flow rate, S stands for the equivalent throttle area. And the air flow rate  $f_{out}$  is a function of crankshaft revolution speed n and the manifold pressure p

$$f_{out}(p(t), n(t)) = (G_{np}n(t) + O_{np})p(t) + G_{nn}n(t) + O_{nn}$$
(2)

where  $G_{np}, O_{np}, G_{nn}$  and  $O_{nn}$  are the air flow rate generation parameters. The equivalent throttle area S is described in terms of the throttle valve angle  $\alpha$  as follows

$$S(\alpha) = a_s \alpha^2 + b_s \alpha + c_s \tag{3}$$

where  $a_s, b_s$  and  $c_s$  are throttle angle / surface conversion parameters. Meanwhile, the manifold pressure and the throttle valve angle should be controlled in a certain range to avoid manifold pressure rising too much for safety reasons [1].

$$0^o < \alpha < 5^o \tag{4a}$$

$$0 \le p \le 1000 \text{ mbar} \tag{4b}$$

In a four-stroke gasoline engine, torque is generated by a piston when it reaches the highest position in the cylinder and the air-fuel mix entrapped is ignited. In this model, torque is assumed constant during the entire expansion stroke. Each transition occurs when the piston reaches one of the dead centers. Engine torque is expressed either with complex polynomials or look-up tables that cover almost every engine speed and manifold pressure range. In our application, since engine speed is limited to a range, we have a limited torque range and therefore we can simplify the model substantially:

$$T = c_m m + c_2 \theta_s \tag{5a}$$

$$m = K_m p \tag{5b}$$

let  $c_1 = c_m K_m$ , and then

$$T = c_1 p + c_2 \theta_s \tag{6}$$

where  $c_1$  and  $c_2$  are constant,  $\theta_s$  is the spark advance angle at the end of intake stroke, corresponding to a bottom dead center. We consider the spark advance angle  $\theta_s$  as the deviation from optimal spark advance, given as a function of the engine working point. The spark advance angle is bounded to avoid knock (too much advance) and misfire (too little advance);

$$0^0 \le \theta_s \le 20^0 \tag{7}$$

For example,  $\theta_s = 0^0$  means that spark coils are programmed to provide the spark at the angular position corresponding to the optimal spark advance.

Finally, the crankshaft model describes the evolution of the crankshaft speed *n*. When the clutch is in the *open* position the two segments of the driveline are disconnected, and the power-train dynamics is given by [1]:

$$\dot{n}(t) = a_{n1}n(t) + b_{n1}(T_g(t) - T_l(t))$$
 (8a)

$$\dot{\theta}(t) = k_c n(t) \tag{8b}$$

where  $\theta$  is the crankshaft angle. In four-cylinder four-stroke engine only one cylinder can be in any one stroke, so only one cylinder is producing torque. Hence, we assume that  $\theta \in [0, 180]$ .  $T_l$  is the load torque acting on the crankshaft, expressed in Nm, and  $T_l \in [0, 12]$ Nm.  $T_g = T - T_p$  represents the effective torque generated by the engine in Nm,  $T_p$  stands for the loss of generated torque due both to pumping and friction. Because the loss torque is very little comparing to load torque, we suppose  $T_p = 0$  [11]. And then we will get  $T_g = T$ . Furthermore,  $k_c$  is the crankshaft angle gain in (8). The dynamic parameters involved in the revolution speed equations are

$$a_{n1} = -30B/\pi J_{eq1}, \quad b_{n1} = 30/\pi J_{eq1},$$
 (9)

where  $J_{eq1}$  and *B* denote the inertial momentum and the viscous friction coefficient of the segment of the power-train from the crankshaft to the clutch.

When the clutch is in the *close* position, the two segments of the driveline are connected, and the power-train dynamics are:

$$\dot{n}(t) = a_{n2}n(t) + b_{n2}(T_g(t) - T_l(t))$$
 (10a)

$$\dot{\boldsymbol{\theta}}(t) = k_c \boldsymbol{n}(t) \tag{10b}$$

where

$$a_{n2} = -30B/\pi J_{eq2}, \quad b_{n2} = 30/\pi J_{eq2}.$$
 (11)

similarly,  $J_{eq2}$  denotes the viscous friction coefficient of the segment of the power-train from the crankshaft to the gear. From (8) and (10), it is very clearly that system parameters have changed when the clutch is in different state (open or close).

According to (1), (8) and (10), the nonlinear system model of engine idle speed are following

$$\dot{p} = a_p G_{np} np + a_p O_{np} p + a_p G_{nn} n + b_p a_s \alpha^2 \qquad (12a)$$

$$+b_p b_s \alpha + b_p c_s + a_p O_{nn},$$

$$\dot{n} = a_{ni}n + b_{ni}c_1p + b_{ni}c_2\theta_s - b_{ni}w, i = 1,2$$
 (12b)

$$\dot{\theta} = k_c n. \tag{12c}$$

where  $a_{ni} = -30B/\pi J_{eqi}$ ,  $b_{ni} = 30/\pi J_{eqi}$ , i = 1, 2 come from (9) and (11), and  $w := T_p + T_l$ , we regard the  $T_p$  and  $T_l$  as a variable w.

In this paper, our control problem is: to design a controller in order to engine speed as close as possible to a certain value  $(n_e = 800\text{RPM})$  when the clutch state changes, meanwhile, the control input constraints (4a) and (7) and the state constraints (4b) should be satisfied. The control inputs are the throttle valve angle and the spark advance angle; the states are engine speed, manifold pressure and crankshaft angle.

## III. QUASI-INFINITE HORIZON NMPC

In this section, a brief summary of quasi-infinite horizon NMPC is given [7] [12].

### A. Problem Setup

We consider the following time-invariant discrete-time nonlinear system

$$x(k+1) = f(x(k), u(k)), \quad x(0) = x_0,$$
 (13)

where  $x(k) \in \mathbb{R}^n$  is state vector,  $u(k) \in \mathbb{R}^m$  is input vector at the discrete-time instant  $k \in \mathbb{Z}_+$  and  $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$  is a nonlinear function.

$$x(k) \in \mathbf{X}, \quad u(k) \in \mathbf{U}, \quad \forall k \ge 0.$$
 (14)

where X denotes the set of feasible states, U denotes the set of feasible control input values. We suppose the system states are completely measurable and the system model is precise known, meanwhile, don't consider external disturbance.

It is assumed in this paper that

(A1)  $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$  is twice continuously differentiable and f(0,0) = 0. Thus,  $0 \in \mathbb{R}^n$  is an equilibrium of the system. (A2)  $U \in \mathbb{R}^m$  is compact, convex,  $X \in \mathbb{R}^n$  is connected.

The point (0,0) is contained in the interior of  $X \times U$ .

Assumption f(0,0) = 0 is not very restrictive, since if  $f(x_s, u_s) = 0$ , we can always shift the origin of the system to  $(x_s, u_s)$ .

In the following, we describe the problem setup for the quasi-infinite horizon nonlinear predictive control scheme introduced in [7] [13].

Problem 1: Find

$$\min_{\bar{u}(\cdot)} J(x(k), \bar{u}(\cdot)) \tag{15}$$

with

$$J(x(k), \bar{u}(\cdot)) = \sum_{i=k}^{k+N-1} \bar{x}^T(i) Q \bar{x}(i) + \bar{u}^T(i) R \bar{u}(i)$$
$$+ \bar{x}^T(k+N) P \bar{x}(k+N).$$

subject to

$$\bar{x}(i+1) = f(\bar{x}(i), \bar{u}(i)), \ \bar{x}(k; x(k), k) = x(k),$$
 (16a)

$$\bar{u}(i) \in \mathbf{U}, i \in \{k, \dots, k+N-1\},$$
 (16b)

$$\bar{x}(i;x(k),k) \in \mathbf{X}, i \in \{k,\dots,k+N-1\},$$
 (16c)

$$\bar{x}(k+N;x(k),k) \in \Omega \subseteq \mathbf{X},\tag{16d}$$

where  $(\bar{x}(k), \bar{u}(k))$  indicate that the predicted values need not and will not be the same as the actual values.  $\bar{x}(i;x(k),k)$ is the predicted trajectory of (13) starting from the actual state x(k) at time k and driven by a given open-loop input function  $\bar{u}$ . N is a finite prediction horizon, (for simplicity of exposition, the control and prediction horizons are chosen to have identical values in this paper),  $Q \in \mathbb{R}^{n \times n}$  and  $R \in \mathbb{R}^{m \times m}$ denote positive definite and symmetric weighting matrices. They are tuning parameters to achieve the desired performance. The positive definite and symmetric matrix  $P \in \mathbb{R}^{n \times n}$ will be determined. The addition constraint (16d) is referred to as terminal inequality constraint that will force the states at the end of the finite prediction horizon to be in some neighborhood  $\Omega$  of the origin, referred to as the terminal region, which will be introduced Subsection III-B.

Now we are able to state the asymptotic stability result of the closed-loop system [12].

Theorem 1: To the nonlinear system (13), suppose that

(a) assumptions A1-A2 are satisfied;

(b)  $Kx \in \mathbf{U}$ , for all  $x \in \Omega$ , the linear feedback controller respects the input constraints in  $\Omega$ ;

(c) the open-loop optimal control problem described by Problem 1 is feasible at time k = 0.

Not considering external perturbation and model errors,

(i) the open-loop optimal control problem is feasible at each time  $k \ge 0$ .

(ii) the closed-loop system is asymptotic stability.

### B. The Terminal Region

The terminal region  $\Omega$  will be chosen such that it is invariant for the nonlinear system controlled by a local state feedback. We consider the Jacobian linearization of the system(13) at the origin

$$x(k+1) = Ax(k) + Bu(k)$$
(17)

If (17) is stabilizable, then a linear state feedback u = Kx can be determined such that  $A_k := A + BK$  is asymptotically stable. For such a given *K*, we can state the following lemma.

**Lemma 1:** Suppose that the Jacobian linearization of the system (13) at the origin is stable. Then,

(a) the discrete Lyapunov equation

$$A_k^T P A_k - P + \kappa Q^* = 0$$

1

admits a unique positive definite and symmetric solution *P*, where  $\kappa > 1$  is a constant,  $Q^* = Q + K^T R K \in \mathbb{R}^{n \times n}$  is positive definite and symmetric.

(b) there exists a constant  $\alpha \in (0,\infty)$  specifying a neighborhood  $\Omega$  of the origin in the form of

$$\boldsymbol{\Omega} := \{ \boldsymbol{x}(k) \in \mathbf{X} | \boldsymbol{x}^T(k) \boldsymbol{P} \boldsymbol{x}(k) \le \boldsymbol{\alpha} \},$$
(18)

such that

(i)  $\Omega \subseteq \mathbf{X}$ , i.e., the state constraints are satisfied in  $\Omega$ ,

(ii)  $Kx \in \mathbf{U}$ , for all  $x \in \Omega$ , i.e., the linear feedback controller respects the input constraints in  $\Omega$ .

(iii) For all  $x \in \Omega$ ,  $x^T P x$  satisfies the inequation in the following

$$x^{T}(k+1)Px(k+1) - x^{T}(k)Px(k) \le -x^{T}(k)[Q + K^{T}RK]x(k)$$

then  $\Omega$  is a terminal region of the nonlinear system (13), u(k) = Kx(k) and  $x^T(k)Px(k)$  are the terminal controller and the terminal penalty function, respectively.

# C. Solving Procedure

The specific algorithm of quasi-infinite horizon NMPC is following

**Step 1** calculate a terminal penalty matrix *P* and a terminal region  $\Omega$  using (17) off-line such that inequality constraint (16d) holds true and the input constraints (16b) and state constraints (16c) are satisfied (see [7]).

**Step 2** at a discrete time instant *k*, solve on-line the optimal problem (15) satisfying (16), according to the current measured variables x(k) and the nonlinear discrete model of (12). Only the first element of the computed optimal control sequence  $\bar{u}(i) \in \mathbf{U}, i \in \{k, ..., k + N - 1\}$  is applied to the system (12).

**Step 3** at the next discrete-time instant k+1, get new state value x(k+1) and return to step 2 to repeat the procedure.

#### IV. APPLICATION TO ENGINE IDLE SPEED CONTROL

In this section, we apply the suggested quasi-infinite horizon NMPC approach to the engine idle speed control.

Considering the parameters of system (12) under different states of the clutch (open and close), the controller consists of controller  $C_1$  and controller  $C_2$  [14] [15]. In every case, only one controller is active. If the the clutch is in the open, the output of the controller  $C_1$  is activated; If the clutch is in the close, the output of the controller  $C_2$  is activated.

$$\mathbf{C} = \begin{cases} C_1 & \text{if } J_{eqi} = 1 \text{kg} \cdot \text{m}^2 \\ C_2 & \text{if } J_{eqi} = 0.1 \text{kg} \cdot \text{m}^2 \end{cases}$$
(19)

where  $C_1$  and  $C_2$  are controllers of the system (12), based the quasi-infinite horizon NMPC approach.

At each instant k, the controller solves the optimal control problem(15) according to current manifold pressure and crankshaft speed, to predict the future optimal engine throttle angle and spark advance angle. The first element of the obtained optimal control sequence as input is applied to the system (12) until the next sampling instant k + 1, at which repeat above the process to find new control sequence for replacing last input, according to new measure states. Our

control goal is to maintain the engine speed at a reference value  $n_e = 800$  RPM.

# A. Solving the Terminal Region and Terminal Penalty Matrix

For the simulation, parameters of model (12) take the following values [1]:

$$\begin{split} a_p &= -1.935 \times 10^7 \mathrm{Pa} \cdot \mathrm{kg}^{-1}, b_p = 4.515 \times 10^9 \mathrm{Pa} \cdot \mathrm{s}^{-1} \mathrm{m}^{-2} \\ O_{nn} &= 5.55 \times 10^{-4} \mathrm{kg} \cdot \mathrm{s}^{-1}, a_s = 1.87 \times 10^{-7} \mathrm{m}^2 \mathrm{deg}^{-2} \\ G_{nn} &= -1.39 \times 10^{-6} \mathrm{kg} \cdot \mathrm{s}^{-1} \mathrm{RPM}^{-1}, b_s = 1.92^{-7} \mathrm{m}^2 \mathrm{deg}^{-1} \\ O_{np} &= -7.78 \times 10^{-9} \mathrm{kg} \cdot \mathrm{s}^{-1} \mathrm{Pa}^{-1}, c_s = 6.143 \times 10^{-6} \mathrm{m}^2 \\ G_{np} &= 1.50 \times 10^{-10} \mathrm{kg} \cdot \mathrm{s}^{-1} \mathrm{Pa}^{-1} \mathrm{RPM}^{-1}, k_c = 6 \mathrm{rad} \cdot \mathrm{RMP}^{-1} \\ c_1 &= 0.0426 \mathrm{Nm} \cdot \mathrm{mbar}^{-1}, c_2 = -0.0548 \mathrm{Nm} \cdot \mathrm{deg}^{-1} \\ \mathrm{When} \ J_{eqi} &= 1 \mathrm{kg} \cdot \mathrm{m}^2, \ \mathrm{then} \end{split}$$

$$a_{ni} = -0.153$$
kg · m<sup>2</sup>s<sup>-1</sup>,  $b_{ni} = 9.554$ RPM · s<sup>-1</sup>Nm<sup>-</sup>

When  $J_{eqi} = 0.1 \text{kg} \cdot \text{m}^2$ , then

$$a_{ni} = -1.53$$
kg · m<sup>2</sup>s<sup>-1</sup>,  $b_{ni} = 95.54$ RPM · s<sup>-1</sup> Nm<sup>-1</sup>

In order to determine a terminal penalty matrix *P* and a terminal region  $\Omega$  for Problem 1, the model (12) is linearized, discretized and normalized at system equilibrium point  $n_e = 800$ RPM,  $p_e = 300$ mbar,  $\alpha_e = 3.45^0$ ,  $\theta_{se} = 5.75^0$ . The sampling time is 0.01s, and then we get

$$x(k+1) = A_i x(k) + B_{ui} u(k) + B_{di} d(k), i = 1, 2$$
(20)

where state variables, control input variables and disturbance variables are normalized.

$$x := \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T = \begin{bmatrix} \frac{p - p_e}{p_e} & \frac{n - n_e}{n_e} & \frac{\theta - \theta_e}{\theta_e} \end{bmatrix}^T \quad (21a)$$

$$u := \begin{bmatrix} u_1 & u_2 \end{bmatrix}^T = \begin{bmatrix} \frac{\alpha - \alpha_e}{\alpha_e} & \frac{\theta_s - \theta_{se}}{\theta_{se}} \end{bmatrix}^T$$
(21b)

$$d := \frac{w - w_e}{w_e} \tag{21c}$$

Equation (20) satisfies input constraints and state constraints as following

$$\frac{0-u_e}{u_e} \le u \le \frac{u_{max}-u_e}{u_e} \tag{22a}$$

$$\frac{D - x_e}{x_e} \le x \le \frac{x_{max} - x_e}{x_e} \tag{22b}$$

As the disturbances in this paper are generally not measured (for example the air conditioning system), there are not disturbances in the predictive model.

Now, we reconsider the system (12). In order to reduce the stability error of engine speed and control energy of throttle angle, the weighting matrices Q and R in objective function (15) are chosen as

$$\mathbf{R} = \begin{pmatrix} 100 & 0\\ 0 & 1 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 1 & 0 & 0\\ 0 & 10 & 0\\ 0 & 0 & 1 \end{pmatrix}$$

We find that the terminal region  $\Omega_1$  of  $C_1$  and  $\Omega_2$  of  $C_2$  are

$$\Omega_1 = \{x \in \mathbb{R}^3 | x^T P_1 x \le 95.9\}, \Omega_2 = \{x \in \mathbb{R}^3 | x^T P_2 x \le 46.8\}$$

where terminal penalty matrix respectively are

$$P_{1} = \begin{pmatrix} 0.0292 & 0.5146 & 0.0284 \\ 0.5146 & 68.2894 & 4.1089 \\ 0.0284 & 4.1089 & 0.4803 \end{pmatrix}$$
$$P_{2} = \begin{pmatrix} 0.3013 & 3.2953 & 0.2860 \\ 3.2953 & 40.56 & 3.751 \\ 0.2860 & 3.751 & 0.4757 \end{pmatrix}$$

### B. Simulation Results

In this section, some simulation results are reported. We assume that the clutch pedal, initially pressed, is released at the instant t = 1s without the external disturbance. The effect of the clutch state (open and close) on the revolution speed of powertrain is shown in figure 2. It is clearly seen that the powertrain speed can almost keep a constant (it is possible that there is a narrow range fluctuation in actual operation) when clutch is in the open position; but, the speed reduces sharply when clutch is in the close position, which may cause engine stall. And then it takes about 3 second to reach stable speed value.



Fig. 2. revolution speed of the engine and state of the clutch

According to the quasi-infinite horizon NMPC methodology above, we design controller  $C_1$  and  $C_2$ . When the clutch is open, we use the  $C_1$  to control the nonlinear system (12); when the clutch is close, we use the  $C_2$  to control the nonlinear system (12) such that the engine speed can maintain the reference value  $n_e = 800$ RPM. The predictive horizon N is 10, initial speed  $n_0 = 300$ RPM, the simulation time is 2.5s.

Now, we suppose that the clutch pedal, initially pressed, is released at the instant t = 1s, without external disturbance. The closed-loop trajectories of engine NMPC in the idle speed mode are shown in figure 3 and figure 4.

As shown in figure 3 and figure 4, although the engine initial speed is very low, but it quickly reaches the reference



Fig. 3. The trajectories of engine speed and throttle angle.



Fig. 4. The trajectories of manifold pressure and spark advance.

point within 1.7s under the operation of  $C_1$  and  $C_2$ . It can be seen that the input constraints (4a) and (7) and state constraint (4b) are not violated.

Finally, we observe the performance of the closed-loop system when adding some external disturbance. In general, the load torque disturbances are caused by the air conditioning system and the steering wheel servo-mechanism, etc. So let the disturbances  $T_l = 6$ Nm and  $T_l = 12$ Nm act on the engine crankshaft respectively, during the time from 2s to 3s. The trajectories of closed-loop system with disturbance are shown in figure 5 and figure 6. When the scope of disturbance torque isn't very large, the engine speed decreases a certain extent value, but the system quickly goes back to operation point. When the disturbance reaches the maximal value, engine speed decreases to 580RPM and keep a period time, after the clutch close, the engine speed returns to ideal value. The system can always operate normally with the maximal disturbance.

In many vehicles, the idle speed is actually controlled with a PID controller. But, there is huge fluctuation in the engine



Fig. 5. NMPC of the engine idle speed with  $T_l = 6$ Nm



Fig. 6. NMPC of the engine idle speed with  $T_l = 12$ Nm

idle speed, due to the system nonlinearity and the lagging of control action. Representative simulation results from [3] are shown in figure 7. Therefore, it is very difficult to obtain good control effect to PID controller. In this paper, significant improvements in terms of idle speed fluctuation and fuel consumption have been achieved with respect to the MPC controller.

## V. CONCLUSIONS

The idle speed control in automotive design is a challenging problem that has been the subject of extensive investigation. In this paper, we focus on 4-cylinder 4-stroke of SI engine system and translate the engine idle speed control problem into optimization problem with input and state constraints. The controller designed using the quasiinfinite horizon NMPC methodology can maintain engine speed as close as possible to the reference value despite change of clutch state and load torque disturbances. The simulation results show that the approach is an effective and promising for the engine idle speed control.



Fig. 7. The trajectories of the engine idle speed with PID

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